Optimization

Task 1: Design the most economical open-topped barrel that will hold 20 L.
The cost of material per cm^2 for the base is triple the cost of the wall.

$V = \pi r^2 h$

$20000 = \pi r^2 h$

$\frac{20000}{\pi r^2} = h$

$SA = \cancel{\pi r^2} + 2\pi r h$ (no top)

$COST = 3\pi r^2 + 2\pi r h$

$COST = 3\pi r^2 + 2\pi r\left(\frac{20000}{\pi r^2}\right)$

$COST = 3\pi r^2 + \frac{40000\pi r}{\pi r^2}$

$COST = 3\pi r^2 + 40000 r^{-1}$

$\frac{dC}{dr} = 6\pi r - 40000 r^{-2}$

$0 = 6\pi r - \frac{40000}{r^2}$

$(r^2)\frac{40000}{r^2} = 6\pi r (r^2)$

$\sqrt[3]{\frac{40000}{(6\pi)}} = r$

$r = 12.85$ cm

$h = \frac{20000}{\pi(ANS)^2} = 38.55$ cm

C' : − (12) + (12.85) − (13)

Task 2: Determine the optimal fare for a bus company to charge if it is determined that each 5 cent increase in fare will result in 40 fewer passengers daily. Currently the company carries 600 passengers per day at a fee of $1.50 each.

fare: $\$1.50 + 0.05n$

passengers: $600 - 40n$

$R = \text{fare} \times \text{passengers}$

$R = (1.50 + 0.05n)(600 - 40n) \rightarrow$ expand...?

$$\frac{dR}{dn} = 0.05(600 - 40n) + (1.50 + 0.05n)(-40)$$

$$0 = 30 - 2n - 60 - 2n$$

$$\frac{4n}{4} = \frac{-30}{4}$$

$n = -7.5 \longrightarrow$ 7.5 price decreases of 5¢ will give the max. revenue.

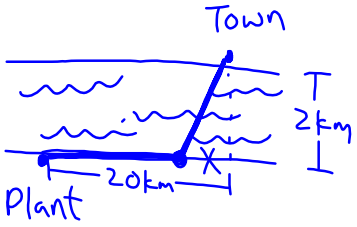
New Price: $\$1.50 - 7.5(0.05) = \$1.50 - 0.375 = \$1.125$

Passengers: $600 - 40(-7.5) = 900$ passengers

Revenue: \$1012.50 (approx \$100/day more)

\$1.10 \$1.15

Task 3: Determine the optimum path of the pipeline from the plant to the town if the cost of installing pipe under water is 4 times the cost of installing pipe underground. It is 2 km across the river and the town is 20 km down-river from the plant.



$hyp = \sqrt{x^2+2^2}$ = underwater pipeline

$20 - x$ = underground pipeline

$X = \pm 0.516\ldots$ km

19.484 km along shore, then 2.06... km across the river.

$$COST = 4\sqrt{x^2+4} + 20 - x$$

$$\frac{dC}{dx} = 4\left(\frac{1}{2}\left(x^2+4\right)^{-\frac{1}{2}}(2x)\right) - 1$$

$$0 = \frac{4x}{\sqrt{x^2+4}} - 1$$

$$1 = \frac{4x}{\sqrt{x^2+4}}$$

$$\left(\sqrt{x^2+4}\right)^2 = (4x)^2$$

$$x^2+4 = 16x^2$$

$$4 = 15x^2$$

$$\frac{4}{15} = x^2$$

$$\pm\sqrt{\frac{4}{15}} = x$$

$$\pm\frac{2}{\sqrt{15}} = x$$

$$\pm\frac{2\sqrt{15}}{15} = x$$

$\pm 0.516\ldots$ km

C': $-$ then $+$ at -0.516 and $+0.516$

0 1

Homework: page 215 #13, 16-18 and worksheet (tomorrow).